# TEACHING NUMERICAL ANALYSIS WITH MATHCAD

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## 1. Introduction

This work is to expound a way to enhance the teaching of Numerical Analysis with technology, especially with PC and math software of Mathcad 2001

### 1.1 Necessity and possibility to change the teaching approach of Numerical Analysis

Numerical Analysis is a course in the curriculum for students of science and engineering. It occupies a special position in the curriculum since it is the course to introduce students how to apply math.--the central subject of all applied math.; it is the course to reinforce and extend students' understanding of Calculus; also, it is the course to require a lot of tedious and repetitive computation. If we say Calculus I is the mark of Math education from static math to dynamic math, then we can say Numerical Analysis is the another mark of Math education from theoretical math to applied math. Hence, when we consider to improve Math education we can not exclude and ignore Numerical Analysis. Numerical Analysis is a different course from the courses students took before. It has its own characters: to employ math ( Calculus, Linear Algebra, Differential Equations, etc) and to implement enormous calculation for numerical methods. These characters determine it requires students' mathematical experience with math and computational skill should be somewhat higher and it need a large amount of time for calculation. Thus, it brings some difficulty to the teaching and learning of Numerical Analysis because of the lack of computational tool before the computer technology became popular. Instructors feel it is hard to teach Numerical Analysis since it is very time-consuming to perform a numerical method and students are not well- prepared. And, students feel it is a hard course and are scared by its requirement. This existing condition in teaching and learning of Numerical Analysis shows: its progress meets the obstruct and it needs improvement. Therefore, the change of teaching approach of Numerical Analysis is on the agenda.
Now, the situation begins to change. First, the demand for using of numerical method to analyse, simulate and design enginerring process and system has been increasing at a rapid rate in recent year. Second, as technologies are growing the amazingly fast computers are commonplace and powerful softwares make it possible to solve highly complex problems. This demand becomes a driving force to make change in teaching and learning of Numerical Analysis and the development of technology provides us with the needed tool and makes it possible to improve the teaching and learning of Numerical Analysis.

### 1.2 Objectives and expected impact for change of teaching approach

The specific objectives set for the change of teaching approach are listed below:
1. To present an approach how to teach Numerical Analysis by using Mathcad;
2. To supply a veriety of relevant problems which are solved with aid of the computer technology as the examples in the teaching of Numerical Analysis;
3. To acquaint students with the potential of modern computer technology for solving numerical problems that may arise in their future profession;

4. To give students an opportunity to hone their skill in programming and problem-solving;
5. To familiarize students with appropriate use of powerful mathematical software;
6. To ease the burden of tedious calculation of students and save time for studying matnenatical aspect of Numerical Analysis.

The expected impacts of the change are as follows:

1. The quality of teaching and learning of Numerical Analysis will be improved;
2. The students' understanding of math will be deepened and their ability to apply math will be enhanced;
3.The students' skill to use computer technology will be sharpened;
4.The students's preparation for problems of Numerical Analysis in their future profession will be raised.

**1.3 Annotation and Acknowledgement**

Although the modern computer technology has enormous potential to enhance the teaching and learning of Numerical Analysis, it still is a problem how to use technology intelligently so that we can achieve desired results. When using technology the case often is that the teaching of technology replaces the teaching of mathematics, i.e., the secondary supersedes the primary so that the mathematics education is not enhanced but damaged. So, we have to handle the problem: what role the technology play in Math education properly.

This work is based on writer's experience from teaching Numerical Analysis and also it is an attempt to find an appropriate way to improve math education with technology.

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## 2. Teaching Bisection Method with Mathcad 2001

### 2.1 About Bisection Method

Bisection Method is an approximate method for rootfinding of non-linear equations. It is a classic method, but simple and effective. Its strategy is to exploit the property of continuous function and the midpoints to approximate the root of an equation.

The positive feature of Bisection method is that the sequence of approximations produced by it is guaranteed to converge and the error tolerance can be prescribed in advance. The negative is that it is slow to converge. Although this is less concerned with speedy computer today, its efficiency still is inadequate.

In this part, I'll introduce two approaches to teach Bisection Method based on different functions of Mathcad 2001. The first one is analogue to hand work but with the aid of the computational functions and the another one is to employ the programing function.

### 2.2 Approach with computational function of Mathcad 2001

This approach is mimic of hand work but with aid of function of software,i.e,The graphic function of Mathcad 2001 will be used to locate the root and the computational functions will be used to estimate the function values and midpoints in the implement of Bisection

Method.

Following example will illustrate the approach.

Given: $2\cdot x\cdot\cos(2\cdot x) - (x+1)^2 = 0$ and I=[-1,0];

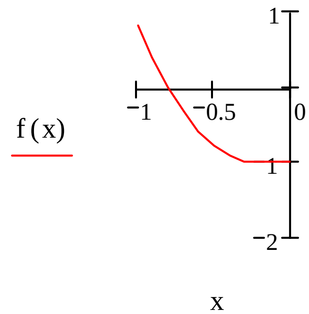
Determine if it has a root in I and find first 3 approximations by Bisection Method if so

Sol'n: First, locate the root by graphing f(x). After graphing, we can determine it exists in I.

$x := -1, -0.9 .. 0$

$f(x) := 2\cdot x\cdot\cos(2\cdot x) - (x+1)^2$

2nd, find 3 approximations: p1,p2,p3 .




$a1 := -1$

$b1 := 0$

$f(a1) = 0.832294$

$f(b1) = -1$

$p1 := \frac{a1 + b1}{2}$

$f(p1) = -0.790302$

$p1 = -0.5$

$a2 := a1$

$b2 := p1$

$p2 := \frac{a2 + b2}{2}$

$p2 = -0.75$

$f(p2) = -0.168606$

$a3 := a2$

$b3 := p2$

$p3 := \frac{a3 + b3}{2}$

$p3 = -0.875$

## 2.3 The approach with program function of Mathcad 2001

In this approach the "For loop" of the program function of Mathcad will be used to perform Bisection Method. Following is the program and an example.

In left program, the input are equation f(x)=0, endpoints of bracket interval [a,b] and accuracy k of $10^{-k}$; the out-put is a table of approximations of root of f(x)=0 in [a,b] and the last one is with accuracy $10^{-k}$. In this program, we use error formula to find n the number of iteration so that we can use "for loop" and sign function to replace $f(p_i)$ so that the conditional statement always works.

$$
Ap(f,a,b,k) := \left|
\begin{array}{l}
s \leftarrow \dfrac{k + \log(b-a)}{\log(2)} \\
n \leftarrow \mathrm{ceil}(s) \\
a_0 \leftarrow a \\
b_0 \leftarrow b \\
\text{for } i \in 0..n-1 \\
\quad \left|
\begin{array}{l}
p_i \leftarrow \dfrac{a_i + b_i}{2} \\
\text{if } \mathrm{sign}(f(p_i)) \cdot \mathrm{sign}(f(a_i)) < 0 \\
\quad \left| \begin{array}{l} a_{i+1} \leftarrow a_i \\ b_{i+1} \leftarrow p_i \end{array} \right. \\
\text{otherwise} \\
\quad \left| \begin{array}{l} a_{i+1} \leftarrow p_i \\ b_{i+1} \leftarrow b_i \end{array} \right.
\end{array}
\right. \\
p
\end{array}
\right.
$$

Now, let's work out an example.
Given: 5cos(2x)-2xsin(2x)=0
and I=[0,1]
Find: its approximate root with
accuracy $10^{-4}$ by Bisection Method
Sol'n: To determine there's a root in I by graph g(x).

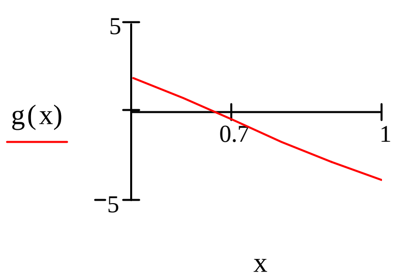
$x := 0.5, 0.6 .. 1$

$a := 0.5$

$b := 1$

$k := 4$

$g(x) := 5\cdot\cos(2\cdot x) - 2\cdot x\cdot\sin(2\cdot x)$




Its exact root is:

$r := \mathrm{root}(g(x), x, 0.6, 0.8)$

$r = 0.656919$

Its approximate root is:

$B := Ap(g, a, b, k)$

$B^T =$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |

## 3. Teaching Interpolation with Mathcad 2001

### 3.1 About Interpolation

Interpolation is the numerical method for constructing the approximate function based on knowledge of its behavior at certain discrete points. Historically, the interpolation problem arose from the need to compute the values for a tabulated function at points not in the table and it was an important task at that time.

Today we seldom have to interpolate for values of sine, logarithm and other such non-algebra -ic functions from tables, since powerful calculators and computers can do that for us. But, the interpolation is still an important subject in Numerical Analysis. This is because that Inter -polation is needed in any field in which measured data are important; this technique can be used to generate function values at points intermediate between these for which measurement are available and it may be used to produce a smooth graph of a function for which values are known only at discrete points, either from measurement or calculation.

Moreover, the computer technology can provide a lot of help in teaching of Interpolation. In this part I'll introduce how to employ the symbolic operation and programing to teach Lagrange and Newton interpolation polynomial.

### 3.2 Approach with symbolic operation of Mathcad 2001

In this part an approach based on the symbolic optration of Mathcad 2001 for the teaching of Lagrange interpolation polynomial will be illustrate by below example.

Example. Given: f(x)=sin(ln(x)) and points: 2, 2.2, 2.55, 3
Find: (1) its Lagrange interpolating polynomial that agrees with it at given points;
(2)use it to approximate f(2.6) and find the error bound;
(3)use it to approximate the integral of f(x) over [2,3].
Sol'n: First, enter the data.

$x0 := 2$

$x1 := 2.2$

$x2 := 2.55$

$x3 := 3$

$f(x) := \sin(\ln(x))$

Second, Determine 4 Lagrange polynomials: L30(x),L31(x),L32(x),L34(x) and construct Lagrange interpolating polynomial P3(x) of order 3.

$$L30(x) := \frac{(x-x1)\cdot(x-x2)\cdot(x-x3)}{(x0-x1)\cdot(x0-x2)\cdot(x0-x3)} \begin{vmatrix} \text{expand} \\ \text{float}, 6 \end{vmatrix} \rightarrow -9.09091\cdot x^{3.} + 70.4545\cdot x^{2.} - 180.545\cdot x + 153.00($$

$$L31(x) := \frac{(x-x0)\cdot(x-x2)\cdot(x-x3)}{(x1-x0)\cdot(x1-x2)\cdot(x1-x3)} \begin{vmatrix} \text{expand} \\ \text{float}, 6 \end{vmatrix} \rightarrow 17.8571\cdot x^{3.} - 134.821\cdot x^{2.} + 334.821\cdot x - 273.21$$

$$L32(x) := \frac{(x-x0)\cdot(x-x1)\cdot(x-x3)}{(x2-x0)\cdot(x2-x1)\cdot(x2-x3)} \begin{vmatrix} \text{expand} \\ \text{float}, 6 \end{vmatrix} \rightarrow -11.5440\cdot x^{3.} + 83.1169\cdot x^{2.} - 196.248\cdot x + 152.38$$

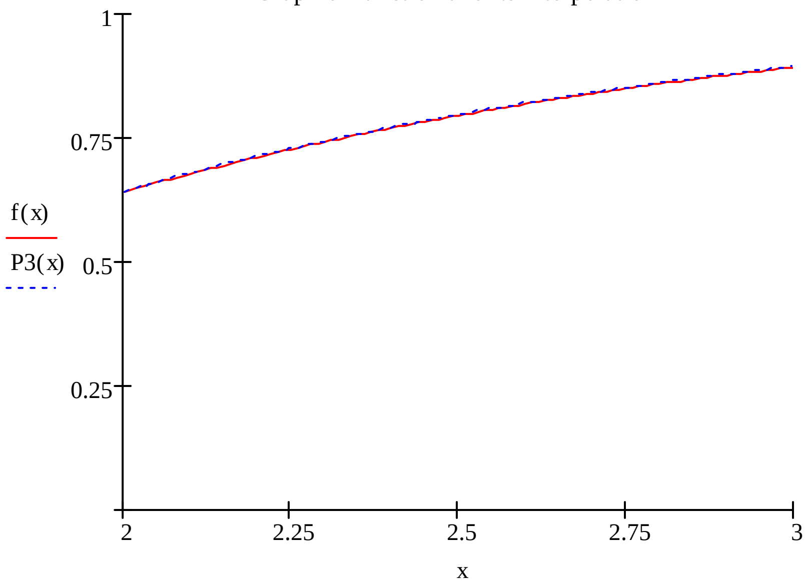
$$L33(x) := \frac{(x - x0)\cdot(x - x1)\cdot(x - x2)}{(x3 - x0)\cdot(x3 - x1)\cdot(x3 - x2)} \left| \begin{matrix} \text{expand} \\ \text{float}, 6 \end{matrix} \right. \rightarrow 2.77778x^{3.} - 18.7500x^{2.} + 41.9722x - 31.166$$

$$P3(x) := f(x0)\cdot L30(x) + f(x1)\cdot L31(x) + f(x2)\cdot L32(x) + f(x3)\cdot L33(x)$$

$$P3(x) \left| \begin{matrix} \text{expand} \\ \text{float}, 6 \end{matrix} \right. \rightarrow 3.474\,10^{-2}\cdot x^{3.} - .3749x^{2.} + 1.4675x - 1.072$$

$x := 2, 2.01 .. 3$

Graph of function and its interpolation




Third, find f(2.6) and estimate error by formula: f(x)-P(x)= $\frac{f^{(n+1)}(\xi)}{(n+1)!}\cdot\prod_{i=0}^{n}(x - x_i)$ .

$P3(2.6) = 0.818706$

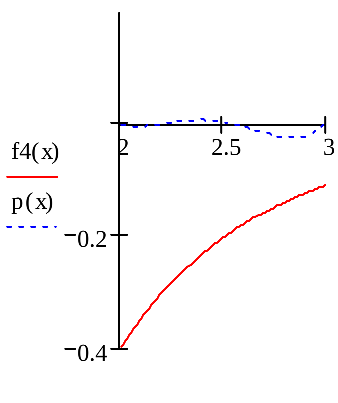
$f(2.6) = 0.816609$

$|P3(2.6) - f(2.6)| = 0.002097$

To find error bound we need to find the absolute max of 4th derivative of f(x) and p(x)=(x-x0)(x-x1)(x-x2)(x-x3) over [2,3]

$$f4(x) := \frac{d^4}{dx^4}f(x) \rightarrow -10\cdot\frac{\sin(\ln(x))}{x^4}$$

$$p(x) := (x - 2)\cdot(x - 2.2)\cdot(x - 2.55)\cdot(x - 3)$$




$f4(2) = -0.399351$

$$p1(x) := \frac{d}{dx}p(x)$$

$x := 2.7$

$r := \text{root}(p1(x), x)$

$r = 2.838552$

$p(r) = -0.024945$

$$B := \frac{|f4(2)| \cdot |p(r)|}{4!}$$

$B = 0.000415$

Finally, use P3(x) to find approximate integral.

$$I := \int_2^3 f(x)\,dx$$

$I = 0.783895$

$$Ia := \int_2^3 P3(x)\,dx$$

$Ia = 0.785876$

**3.3 Approach with programing and symbolic operation of Mathcad 2001**

In this part the approach with programming and symbolic operation for teaching of Newton interpolation polynomial will be illustrated by following example.

Example.Given: f(1)=0.7651977, f(1.3)=0.620086, f(1.6)=0.4554022, f(1.9)=0.2818186, f(2.2)=0.1103623. Construct interpolating polynomial of degree 4 by Newton's interpolatory divided difference formula and use it to approximate f(2).

Sol'n: First, enter the data:

$a_0 := 1$

$a_1 := 1.3$

$a_2 := 1.6$

$a_3 := 1.9$

$a_4 := 2.2$

$b_0 := 0.7651977$

$b_1 := 0.620086$

$b_2 := 0.4554022$

$b_3 := 0.2818186$

$b_4 := 0.1103623$

Second, use following program to figure out divided-differences. In following program, use a for-loop to enter above set of data, and then use double for-loop to figure out the divided-differences. The output of the program is a matrix and the elments on the diagonal are the divided-differences needed in Newton's formula.

```
f := | for  i ∈ 0..4
     |    | x_i ← a_i
     |    | f_(i,0) ← b_i
     | for  i ∈ 1..4
     |   for  j ∈ 1..i
     |     f_(i,j) ← (f_(i,j−1) − f_(i−1,j−1)) / (x_i − x_(i−j))
     | f
```

$$f = \begin{pmatrix} 0.765198 & 0 & 0 & 0 & 0 \\ 0.620086 & -0.483706 & 0 & 0 & 0 \\ 0.455402 & -0.548946 & -0.108734 & 0 & 0 \\ 0.281819 & -0.578612 & -0.049443 & 0.065878 & 0 \\ 0.110362 & -0.571521 & 0.011818 & 0.068069 & 0.001825 \end{pmatrix}$$

Then, use Newton's interpolatory divided-difference formula to construct polynomial.

$$p4(t) := f_{0,0} + \sum_{i=1}^{4} \left[ f_{i,i} \cdot \prod_{k=0}^{i-1} \left( t - a_k \right) \right]$$

$$p4(t) \left| \begin{matrix} \text{simplify} \\ \text{float}, 6 \end{matrix} \right. \rightarrow .977735 + 7.33913 \cdot 10^{-2} \cdot t - .343047 \cdot t^{2.} + 5.52928 \cdot 10^{-2} \cdot t^{3.} + 1.82510 \cdot 10^{-3} \cdot t^{4.}$$

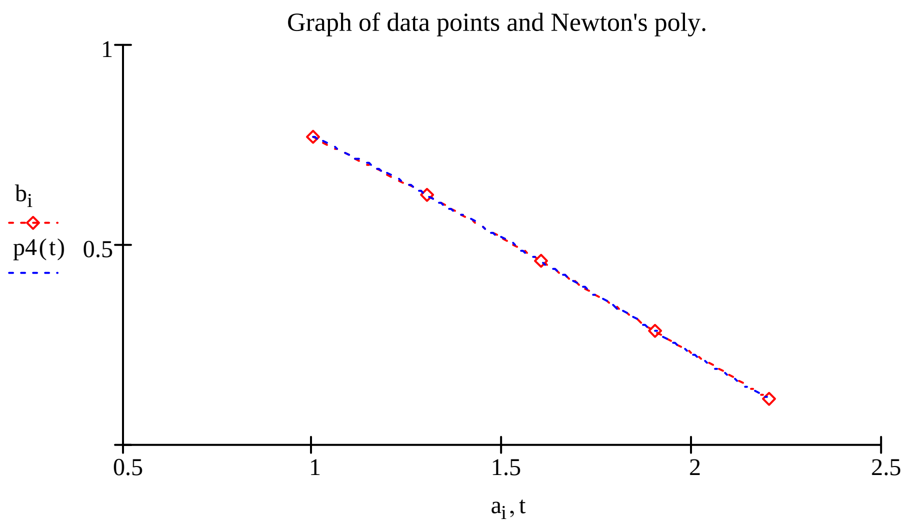
Then, find the approximate value of f(2) is

$p4(2) = 0.223875$

Finally, Plot the data points and Newton's interpolation polynomial in a graph below:

$i := 0..4$

$t := 1, 1.01..2.2$




## 4. Teaching Numerical Method of ODE with Mathcad 2001

### 4.1 About Numerical method of ODE

Ordinary differential equations are important mathematical model which are used to model the problems in science and engineering. Any textbook of Differential Equations details a number of methods for explicitly finding their solutions. However, in most real life situation, the differential equation that models the problem is too complicated to solve exactly. Thus, we often need to approximate its solution and the numerical methods become an important part of Numerical Analysis.

In this part I'll introduce two approaches to teach numerical methods of ODE. One is to use the program function of Mathcad 2001 and the another one is to use seeded iteration. And, I'll choose the Runge-Kutta method of order 3 as the numerical method.

### 4.2 Approach with program function of Mathcad 2001

In this section I'll illustrate the approach with program function of Mathcad 2001 through an
example with Runge-Kutta method of order 3.

Program:

$$NS\_RK3(f,a,b,y0,h) := \left| \begin{array}{l} n \leftarrow \dfrac{b-a}{h} \\ x_0 \leftarrow a \\ y_0 \leftarrow y0 \\ \text{for } i \in 0..n-1 \\ \quad \left| \begin{array}{l} x_{i+1} \leftarrow x_i + h \\ k1 \leftarrow h \cdot f(x_i, y_i) \\ k2 \leftarrow h \cdot f\left(x_i + \dfrac{h}{2}, y_i + \dfrac{k1}{2}\right) \\ k3 \leftarrow h \cdot f\left(x_i + \dfrac{3 \cdot h}{4}, y_i + \dfrac{3 \cdot k2}{4}\right) \\ y_{i+1} \leftarrow y_i + \dfrac{2 \cdot k1 + 3 \cdot k2 + 4 \cdot k3}{9} \end{array} \right. \\ p \leftarrow \text{augment}(x, y) \\ p \end{array} \right.$$

Remark on program:
In left program,
input are
f:: function of ODE
a: initial value of x and left endpoint of interval
b: right endpoint of interval
y0: initial value of y
h: step size

output is a table which includes numerical solution over interval with corresponding x-values. .

Now, let's work out an example.

Given: y'=1+xsin(xy) ,y(0)=0 and interval [0,2]
Find: its numerical solution over I by Runge-Kutta method of order 3 with step size h=0.1
Sol'n:

$f(x,y) := 1 + x \cdot \sin(x \cdot y)$

$a := 0$

$b := 2$

$y0 := 0$

$h := 0.1$

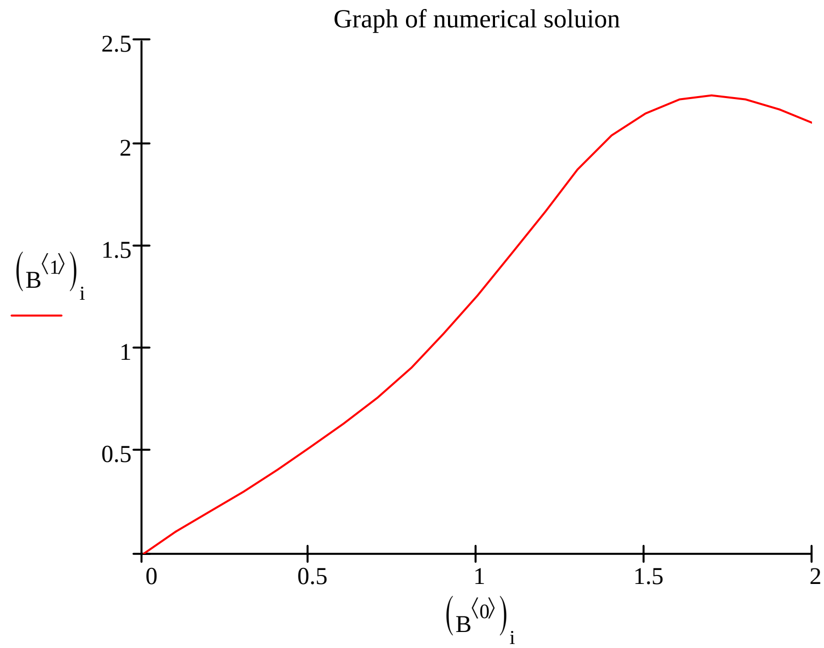
The numerical solution over I is the 2nd row of

$B := NS\_RK3(f,a,b,y0,h)$

$B^T =$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |

$i := 0..20$




## 4.3 Approach with seeded iteration of Mathcad 2001

In this section I'll describe how to use seeded iteration to perform Runge-Kutta method of order 3 to find numerical solution of IVP of 1st order ODE by following example.
Given: IVP y'=1+xsin(xy), y(0)=0 and interval [0,2]
Find: its numerical solution over I by seeded iteration with RK3 with step size h=0.1
Sol'n:

$f(x,y) := 1 + x\cdot\sin(x\cdot y)$

$a := 0$

$b := 2$

$h := 0.1$

$n := \frac{b-a}{h}$

$n = 20$

$k := 0..n$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} := \begin{bmatrix} x_k + h \\ y_k + \frac{h}{9}\cdot\left(2\cdot f(x_k,y_k) + 3\cdot f\left(x_k + \frac{h}{2}, y_k + \frac{h}{2}\cdot f(x_k,y_k)\right) \ldots \atop + 4\cdot f\left(x_k + \frac{3\cdot h}{4}, y_k + \frac{3\cdot h}{4}\cdot f\left(x_k + \frac{h}{2}, y_k + \frac{h}{2}\cdot f(x_k,y_k)\right)\right)\right) \end{bmatrix}$$

$k := 0..20$

$x_k$ =

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$y_k$ =

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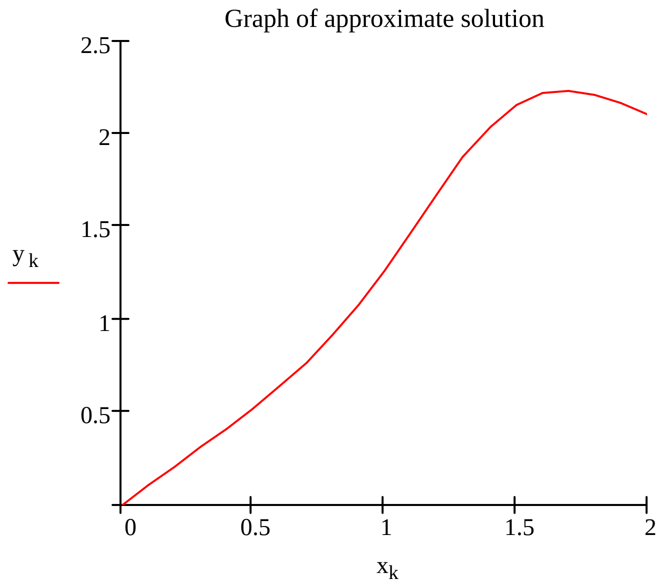



Remark: This is the approach of seeded iteration. The matrix of x0 and y0 is the seed and the matrix of xk+1 and yk+1 is the iterative formula of RK3. The approximation grows from the seed and iterative formula. This seeded iteration can stead of a progam of for loop. It is simpler than the program. So, seeded iteration is one of powerful function of Mathcad.

**5. PC approach for solving problem in Numerical Analysis**
**5.1 About solving problem in Numerical Analysis**

It is not easy to work out a problem in Numerical Analysis because it often involves a lot of difficult aspects and requires not only mathematics background but also other skill to solve it. Now, the computer technology provides us with a powerful tool to overcome these difficulty.

In this part I'll describe an approach how to use PC to solve the problem in Numerical Analysis through an example--fixed point iteration.

**5.2 Sample approach**

In this section a problem of fixed point iteration will be used to illustrate the PC approach to solve the problem in Numerical Analysis.

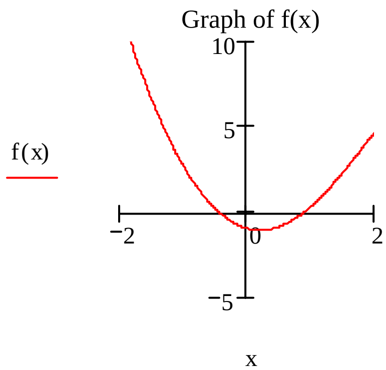
Fixed point iteration is often used to find the approximate solution of an equation.The procedure to perform a fixed point iteration includes following steps: To locate the solution of the equation; To determine a function of fixed point to convert the rootfinding problem to fixed point problem; To implement fixed point iteration to get desired approximate solution. In this procedure every step is not easy. To locate the solution is difficult; To determine an appropriate function of fixed point is difficult; to implement fixed point iteration which needs a lot of calculation is difficult. But, now the computer technology can ease the job. I'll use following example to illustrate.

Given: equation $3x^2 - e^x=0$; Determine a function g(x) and an interval [a,b] on which fixed point iteration will converge to the positive solution of given equation with accuracy $10^{-5}$.

Sol'n: First, To locate the positive solution by graph the function.

$x := -2, -1.99 .. 2$

$f(x) := 3 \cdot x^2 - e^x$




From graph we can see f(x) has a positive solution in [0,1]

Second, To determine a function g(x) which fixed point is the solution of f(x)=0 and an interval on which fixed point iteration converges to the solution. This is pretty tough job. But, from Mathematics we know: the g(x) and [a,b] are proper if they satisfy 3 conditions: g(x) is continuous on [a,b]; $a \le g(x) \le b$ and $\left|\frac{d}{dx} g(x)\right| \le k.<1$ . And, PC can help us to complete the job.

From f(x)=0 we can get 3 g(x), they are g(x)= $\frac{1}{\sqrt{3}} \cdot e^{\frac{x}{2}}$ , g(x)= $\frac{e^x}{3x}$ and g(x)=ln(3 $x^2$).

Which one is proper? It should satisfy 3 conditions mentioned above. How to verify if a g(x) satisfies these conditions? PC can help us to check if a g(x) satisfies these conditions through graphing g(x), g'(x) and line y=x. since fixed point is the intersection point of y=g(x) and y=x;

an proper interval must contain this point and on which g(x) is continuous, $a \le g(x) \le b$ and |g'(x)|<1.i.e. looking for such g(x) which graph and intersection point with y=x is located in a square having y=x as diagonal and |g'(x)|<1.

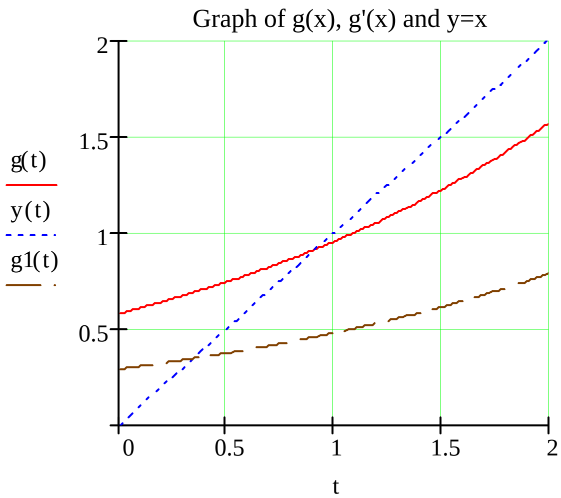
Through graphing these 3 g(x), we can determine the first one is proper choice. Below graph shows this fact.

$t := 0, 0.01 .. 2$

$g(t) := \frac{1}{\sqrt{3}} \cdot e^{\frac{t}{2}}$

$g1(t) := \frac{d}{dt} g(t) \rightarrow \frac{1}{6} \cdot 3^{\frac{1}{2}} \cdot \exp\left(\frac{1}{2} \cdot t\right)$

$y(t) := t$




g(0) = 0.57735027
g(1) = 0.95188967
g1(0) = 0.28867513
g1(1) = 0.47594483

From above graph, we can see the fixed point is in [0.5,1], so intervals [0,1], [0,2] and [0.5,1] all are proper interval.we can choose. say, we use [0,1]. Also,g(x) and g1(x) are increasing on [0,1], so g(0)=0.577 is min, g(1)=0.952 is max, thus 0<g(x)<1; g1(0)=0.289 is min , g1(1)=0.476 ix max, thus |g'(x)|<1. Therefore, such g(x) is our desired.
Third, To determine the number of steps needed for desired accuracy and perform fixed point iteration to get approximate solution. Let n be the number of steps, k is max of |g'(x)|, a initial guess and m indicate accuracy, then from error formula of fixed point iteration we have: n> $\frac{\log(1-k)-\log(|g(a)-a|)-m}{\log(k)}$. And, we use a program to perform the iteration.

$a := 0.5$
$k := 0.5$
$m := 5$

$$s := \frac{\log(1-k)-\log(|g(a)-a|)-m}{\log(k)}$$

$s = 15.55873412$

```
p(a,g,n) := | p_0 <- a
            | for i ∈ 0..n-1
            |   p_{i+1} <- g(p_i)
            | p
```

Number of steps:
$n := \text{ceil}(s)$
$n = 16$

The exact solution:
$r := \text{root}(f(x), x, 0, 1)$
$r = 0.91000757$

The approximate solution is
$B := p(a,g,n)$

$B^T =$

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |

## 6. Summary

Three years ago I started to change the teaching approach of Numerical Analysis. Now, it might be the time to make a look back and brief summary for the practice in these three years. I will summarize from three aspects which are closely related to the change of teaching approach.

They are computer technology, students and instructor.

### 6.1 Mathcad and change of teaching approach

The change of teaching approach is featured by integrating computer technology with the teaching and learning of Numerical Analysis. Therefore, to select a computer technology is the first problem needed to be solve.

The development of computer technology provides us with a variety of choices. In hardware, there're Laptop, Desktop PC, Workstation and Mainframe computer. In software, there're scientific programming language such as Fortran, C, C++; software package for specific problem, such as ALPACK, EISPACK; and Computer Algebra Systems such as MatLab, Maple, Mathematca and Mathcad.

Now, the PC is popular day by day and slowly becomes a part of daily life, moreover, modern PC is so power-increased that it is good enough for teaching and learning of Numerical Analysis.

CAS(computer algebra syctem, this name seems not too descriptive)is a kind of fully interactive mathematical software system. It possesses abilities: performing algebraic operations, carrying out calculus operations and programming. It can be used to get numerical, symbolic and graphic solution of mathematical problems. Such features and abilities of CAS are specially useful for Numerical Analysis and make it a prefered tool for teaching and learning of Numerical Analysis.

Mathcad is one of popular CAS in the world. Besides it has common abilities and features of CAS, it has some exclusive features specially useful for Numerical Analysis, for example, its "seeded iteration" is simple and useful since a lot of numerical procedure is iterative and it can replace a program. Another example is its words processor function, its words processor is much better than other CAS and can be regarded as scientific words processor which can integrate text, equations and graphes to simplify documentation.

For change of teaching approach I integrated PC and Mathcad into the teaching of Numerical Analysis.

### 6.2 Students and change of teaching approach

In a teaching and learning process there're two sides: students and instructor. Any change in teaching approach will certainly affect students and their reflection is an important aspect to check the consequence of the change of teaching approach.

In this part I'll summarize positive and negative consequence of the change of teaching approach which are based on students' feedback: their evaluation of course and my talk with them.

**Positive consequence**

1. Most of students welcomed the change of teaching approach and were interested in learning computer technology. In two classes I taught in past three years there're more than 75% of stydents bought their own PC for the course. This fact showed they supported the change by action and were interested in computer technology.
2. Most of students thought they learned more computer technology in new approach than old approach.

3. A lot of students reflected they got intuitional impression of the power of modern computer technology and said they were astonished.
4. Most of students felt their ability using computer technology to solve math problem was increased.
5. Almost all students agreed this approach lightened their burden of enormous operations and saved them much time in completing assignments.
6. A lot of students thought their preparation for Numerical Analysis problems in their future professions became better than before.

**Negative consequence**

1. Some of students thought this approach made Numerical Analysis more difficult since it added new requirement: computer technology.
2. Some of students thought they could not take Numerical Analysis course any more since they didn't have computer.
3. Some of students thought the computer technoloy was a torture and felt frustrated since the computer technology made them less, not more, productive.
4. Some of students thought they don't need to study Numerical Analysis any more and the only thing they need to study is the computer technology since the software was so powerful and versatile that it can solve all math problem.

**6.3 Instructor and change of teaching approach**

In this part I'll look back and summarize my practice in these three years from the point of view of an instructor who conducts the change of teaching approach. Generally speaking, in the change of teaching approah something is successful and something is not, and there're some problems need to be solved.

**Successful aspect**

1. I think after changing of teaching approach the quality of teaching and learning of Numerical Analysis is somewhat advanced. The quantity and quality of the course work are increased and the performance of most students is better than before.
2. I think after changing of teaching approach students' understanding of computer technology is deepened and their interesting is rised.
3. I think after changing of teaching approach students learn more knowledge of computer technology and their computational skill is improved.
4. I think after changing of teaching approach students' ability to use computer technology to solve math problems is strengthened and their preparation for future profession is improved.

**Existing problems**

1. Students' background of computer technology is not sufficient. The computer experience most of students have is limited to playing computer game, surfing internet, using Window and words processor. They don't have experience how to use math softwares. This fact forced me to spend one third of class time to teach how to use Mathcad and hurted the teaching of math aspect of Numerical Analysis.
2. The math part of Numerical Analysis is not enhanced as expected because a lot of time is spent to teach computer technology.
3. A trend to ignore the learning of math part of Numerical Analysis occurs. Some of students think: the computer technology is so powerful and almost is versatile; the only thing they need to study is computer technology and pay few attention to the learning of math part of Numerical Analysis.
4. The examination system needs to be developed. Since this approach adds a new factor to the math course, how to evaluate the performance of students and what criteria can be

used for evaluation become a problem, especially for the cases: students' performance in math is good and in computer technology is not good or conversely. The way I adopt now is to divide tests and exams into two parts: handwork part in classroom and computerwork part as homework. Often there're some problems in computerwork part.

**List to do**

1. In order to enhance students' background of math sofyware I plan to suggest a prerequisite course of computer technology.
2. In order to let every student who registers in the course have a computer use I plan to apply for a grant to buy some Laptop to lend to students.
3. In order to help students to overcome the "initial drop"problem which often happens when we learn to use a new tool and frustrates students, I plan to develop some instructional material.
4. In order to correct students' misunderstanding of computer technology I plan to enhance the teaching of math part of Numerical Analysis and emphasize: a computer essentially is dumb and must be given detailed and complete instruction for every step it is to perform and these instructions are based on math.

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